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ABSTRACT
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understandings and skills that are involved in being able to construct
graphical representations of data and to interpret these graphs. The study
examined four components of prior understanding required for graphic
representation that were adapted from a learning map from the Atlas of
Science Literacy (American Association for the Advancement of Science, 2001).
The components involved knowledge about: (1) coordinate relationships; (2)
graphs showing a variety of relationships; (3) reading simple tables; and (4)
graphic display. Complete data from 82 sixth graders from the classes of 3
teachers were collected and analyzed using multiple regression, factor
analysis, and structural equation modeling (SEM) to examine the nature and
alignment of assessment items that could be used to measure these components
of prior knowledge. The results indicate that the SEM models reflect
significant fits with the Atlas map. While further research with a larger
sample will be needed to examine the relationships among the four components
and with the target learning goal for graphic representation, the SEM
analysis appears to be a promising approach for modeling the construction of
student knowledge in specific content areas of mathematics. Six appendixes
contain some assessment items and student responses. (Contains 38
references.) (Author/SLD)

Running head: INVESTIGATING MATHEMATICAL COMPLEXITIES

Investigating the Complexity of Middle Grade Students' Understandings of

Mathematical Constructs: An Example from Graphic Representation

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KEY WORDS: "MATHEMATICS UNDERSTANDING", "MODELS OF

LEARNING", "GRAPHIC REPRESENTATION"

Abstract

This study explored a model for students' development of the understandings and skills that are involved in being able to construct graphical representations of data and to interpret these graphs. Specifically, the study examined four components of prior understanding required for graphic representation that were adapted from a learning map from the *Atlas of Science Literacy* (AAAS, 2001). The components involved knowledge about: coordinate relationships, graphs showing a variety of relationships, reading simple tables, and graphic display.

Complete data from 82 sixth graders from the classes of three teachers were collected and analyzed using multiple regression, factor analysis, and structural equation modeling (SEM) to examine the nature and alignment of assessment items that could be used to measure these components of prior knowledge. The results indicate that the SEM models reflect significant fits with the *Atlas* map. While further research with a larger sample will be needed to examine the relationships among the four components and with the target learning goal for graphic representation, the SEM analysis appears to be a promising approach for modeling the construction of student knowledge in specific content areas of mathematics.

Investigating the Complexity of Middle Grade Students' Understandings of

Mathematical Constructs: An Example from Graphic Representation

This paper addresses the challenge of describing students' development of important mathematics concepts and skills. In this study we focus on the alignment of learning goals and achievement, with special attention to the prior knowledge and conceptions that must be present or developed. We seek an understanding of the relative importance of these understandings in modeling and predicting trajectories of achievement. Specifically, the study explores a model for students' development of the understandings and skills that are involved in being able to construct graphical representations of data and to interpret these graphs.

Theoretical Framework

Teaching and learning mathematics with understanding involves some fundamental forms of mental activity: (1) constructing relationships, (2) extending and applying knowledge, (3) reflecting about experiences, (4) articulating what one knows, and (5) making knowledge one's own (Carpenter & Lehrer, 1999). Furthermore, the specific classroom activities and teaching strategies that support these mental activities, include appropriate tasks, representational tools, and normative practices that engage students in structuring and applying their knowledge and in reflection and encourage articulation about tasks and about their own mental activities. Fairness and equity is an especially important issue in making learning and understanding available for all students. There may be differential effects of this type of instruction for some students (Secada & Berman, 1999).

There are specific instructional and learning factors that produce cognitive change and understanding of mathematics concepts and procedures in middle grades students. In particular, activities that (1) build on students' prior ideas about mathematics and (2) promote student thinking and reasoning about mathematics concepts are important for building understanding. Research on these variables supports their importance in mathematics teaching and learning that is designed to lead to conceptual change (Posner, Strike, Hewson, & Gertzog; 1982).

Building on Student Ideas about Mathematics: The importance of taking account of students' ideas has long been recognized. Ausubel (1968) noted that "the most important single factor influencing learning is what the learner already knows." If students have narrow conceptions and representations of ideas or procedures that do not extend to other situations, their subsequent work can result in misconceptions (Fischbein, Deri, Nello, & Marino, 1985; Bell, Greer, Grimison, & Mangan, 1989). For example, a misconception common among junior high as well as college students is the interpretation of a graph as a picture (Ben-Zvi & Arcavi, 2001; Berg & Smith, 1994; Elby, 2000).

Teachers who understand students' knowledge and thinking are able to use this information to improve the quality of their instruction (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Cobb et al., 1991). Several strategies have been found that are effective in identifying and addressing prior knowledge. For example, a discussion of how students perceive the difference between two solutions to an exercise or problem can provide insights into student understanding (Cobb, 1988). Also, an assessment of how students extend procedures to other contexts and situations can reveal misconceptions or lack of understanding (Hiebert & Wearne, 1986). Both of these strategies apply to a wide

range of mathematical ideas and procedures. Unless instruction attends to students' prior knowledge and teachers are alerted to it, the sequence of activities might be inappropriate (Mack, 1990).

Graphic Representation Skills and Concepts: Results from the National Assessment of Educational Progress (NAEP) have not changed much from 20 years ago when it was reported that students in grades 4 and 8 generally do well in simply reading information from graphs, but perform poorly when asked to combine two or more pieces of information to answer a question (Bestgen, 1980). Performance on data extraction tasks depends on the type of display used, the person's experience, and the complexity of the graphical display (Meyer, Shinar, & Leiser; 1997). One of the early attempts to examine the effect of prior knowledge on comprehending graphs was to apply a schema-theoretic perspective. Mathematics and reading achievement, along with three subtests of prior knowledge (Topic, Mathematical Content, and Graphical Form) were used as predictors of graphical comprehension. For 7th graders, mathematics and reading achievement were the best predictors; the knowledge of mathematical content of graphs added somewhat; and prior knowledge of topic and graphical form were poor predictors for grade 7 (Curcio, 1987).

Little is known about how children go about constructing and analyzing data and drawing inferences from it. Lehrer and Romberg (1996) used a classroom-based "design experiment" to explore fifth graders' data modeling processes. The authors concluded that students need opportunities to construct and develop ideas and skills in data modeling. At the same time, it seems important to develop models for how these skills and concepts develop.

The present exploratory study considers questions about how middle school students develop understandings and skills necessary for constructing and interpreting graphic representations. This paper reports initial progress toward developing a model for how students' graphic representation skills and concepts develop in complexity. The following questions were addressed:

1. What prior knowledge and skills can be used to predict sixth graders' knowledge about graphic representation and interpretation?

2. How can the development of sixth graders' knowledge and skills in graphic representation and interpretation be modeled?

Methods

Four sixth grade teachers in a suburban middle school agreed that the curriculum *Connected Mathematics* (CMP) would address their desire to provide better instruction, especially for students who had not achieved well on the state mathematics assessment. Three of the teachers ($n=3$) and their students in five classes ($n=140$) agreed to participate in the data collection part of the project, beginning in the Fall, 2000. The agreed purpose is to study mathematics teaching and learning, not to compare individual teachers. Within each class, 3 students ($n=15$) were identified for closer study with the intent of developing specific trajectories and mental models of their learning of specific mathematics concepts. These students will be followed for 3 years, beginning in grade 6. In each year, teachers will be added to the study.

The study simultaneously employs two designs: (1) Case studies of instructional implementation of CMP materials by the three middle grades mathematics classroom teachers and individual students, and (2) Study of students' performance and achievement

on specific mathematical learning goals (TEKS standards) as it relates to effectiveness in classroom implementation. This report will focus on the second of these by analyzing quantitative student achievement data.

An example of a learning model was developed to reflect conceptualization of students' learning. The model was adapted from the Graphic Representation "strand map" of Project 2061's *Atlas of Science Literacy* (AAAS, 2001). The map displays the ideas and skills that contribute to an understanding of the topic of graphic representation and shows how the ideas and skills relate to each other and how they progress from one grade level to the next.

INSERT FIGURE 1 HERE

Design and Data Sources. Student learning and achievement of graphic representation concepts and skills were assessed through multiple methods and strategies including unit tests, performance tasks from the Balanced Assessment (2000), interviews with selected students, a spring Benchmark Test (TAAS pretest), and end of year Texas Assessment of Academic Skills (TAAS) mathematics test, which is the state assessment. For the analyses reported here, specific items on Graphic Representation were used from (a) the spring Benchmark Test, (b) an open-ended Balanced Assessment task (Vet Club), and (c) the 2001 TAAS sixth grade mathematics test. See Appendixes A through D for examples of the spring Benchmark Test and Appendix E and F for the Vet Club task and a sample of a student response.

The data were analyzed using a three-phased approach. First, a multiple regression analysis was used to determine how well the target graphic representation

learning goal was predicted by prior knowledge. Next, a factor analysis was conducted using the four prior knowledge components as *a priori* factors, in order to confirm that these components were indeed appropriate to proceed with the final Structural Equation Modeling (SEM) analysis. Finally, based on the supportive evidence from the first two analyses, an SEM analysis was carried out to examine the fit of the theoretical and measured models of students' graphic representation knowledge and skill development.

MultipleRegression. We performed a multiple regression analysis with students' performance of graphic representation as the dependent variable and the measured variables for each of the objectives specified in the theoretical map (See Figure 1) as independent variables.

Factor Analysis. A factor analysis was conducted to determine if the theoretical constructs were evident in the data prior to submitting the data to further analyses in SEM. The number of factors was set to four, *a priori,* in accordance with theory articulated in Figure 1. From the correlation matrix the data were considered to be too highly correlated to meet of the case for orthogonal rotation, hence, a promax rotation with Kaiser normalization was used. Thompson (1997) argued that when obliquely rotating data it is important to interpret both the factor pattern matrix and the factor structure matrix to ascertain the overall structure. Although this is true and necessary, we only considered the rotated structure matrix because we were following the analysis with SEM for additional rigor.

Structural Equation Modeling. It was important to determine if the theoretical learning map (Figure 1) is a true model of student learning of graphic representation. A structural equation model (SEM) was used to investigate this correspondence (Klem, 2000). As

Thompson (1991) noted, "Multivariate methods best honor the reality to which the researcher is purportedly trying to generalize." (p. 80) Canonical correlation analysis (CCA) has been found to subsume all other parametric analyses such as t-tests, ANOVA, regression, and X^2 (Frederick, 1999; Knapp, 1978). In seminal work Jöreskog and Sörbom (1979) indicated that SEM is an even more general case of the GLM, subsuming all other cases of the as special cases (Capraro & Capraro, 2001; Knapp, 1978; Thompson, 1991). SEM has been termed "the single most important contribution of statistics to the social and behavioral sciences during the past twenty years" (Lomax, 1989, p. 171). Similarly, Stevens (1996) argued that SEM is ". . . one of the most important advances in quantitative methodology in many years" (p. 415).

Because SEM subsumes all other parametric statistical analyses it provides some interesting options for the researcher. First, all other analyses (e.g., *t*-tests, ANOVA, regression, MANOVA, and CCA) can be conducted as special cases in SEM. This is of conceptual interest but is often not a practical shortcut or an elegant solution that efficiently answers the research question at hand. However, when the research questions deal with understanding the underlying structure (EFA) of a set of items or in confirming a theory from a set of data (CFA) SEM can provide truly elegant and unique solutions (Stevens, 1996).

In SEM, the models include unobservable variables identified as latent constructs, defined by observed variables, which by theory fit the construct(s). Measurement error, reflecting score reliability is also typically estimated as a unique and an essential part of SEM analyses (Jöreskog, 1969, 1970, 1973, 1977; Jöreskog, & Goldberger, 1975; Jöreskog, & Sörbom, 1978). SEM's major advantage over other analytic methods is that

it accounts for measurement error. A major distinguishing feature of SEM is that score reliabilities are estimated as part of structural modeling. Thus, structural models test both substantive hypothesis and measurement models. This may not be obvious to many applied researchers, because rather than estimating reliabilities directly, SEM estimates error variances instead. In the present study, the data from measured variables are used to confirm hypothesized relationships among learning map constructs as shown in Figure 1 account for student achievement.

In the SEM analysis, several competing models are considered in an attempt to achieve an identified model. The data was analyzed using AMOS 4.0, which provides several fit statistics. The most general measure of overall fit is the Chi-square statistic which is sensitive to sample size. Other statistics that are considered in obtaining the best model include the RMSEA index, Modification Indices (MI), Normed Fit Index (NFI), Critical Ratio (CR) and the Wald Statistic (Thompson, 2000).

Results

The overall achievement results at the end of the first year, as measured by the TAAS mathematics test were very good in that 222 of 225 students (99%) passed the test. Of special note was only one minority and/or second language students failed, compared with 30 - 35% failure rate in previous years.

To explore the extent to which student prior knowledge predicted performance on Graphic Representation, a linear regression model was applied, using fall semester measures of understanding and working with data as predictors of the TAAS Graphic Representation objective. As shown in Table 1, this analysis yielded an R^2 of 46.5%

indicating a modest development of student understanding half-way through the first year.

INSERT TABLE 1 HERE

The results of the Component Factor Analysis shown in Table 2 seemed to support the underlying construct, since the factor structure coefficients supported the overall understanding of student learning as depicted in Figure 1. The number of subjects with complete data was too small to be confident that there cannot be other possible interpretations that can fit the data. Nonetheless, it appears that the four prior knowledge components in the theoretical model (Figure 1) were measured by the items that we identified. The four prior knowledge factors accounted for 48% of the variance in graphic representation knowledge.

INSERT TABLE 2 HERE

In order to explore the complexities involved in students' understanding and skills, four separate SEM models were developed, each one representing one of the four components of the Graphic Representation map as a focus.

SEM Results

A critical question is how to determine whether an *a priori* model fits the data or how to compare the relative fit of the model. Because of relatively strong theory, no competing models were selected. The Cronbach's (1951) alpha coefficient for Vet Club was .74, and the Spring Benchmark was .82. Whereas tests of statistical significance and indexes of fit aid in the evaluation of the fit of a model, there is ultimately a degree of subjectivity and professional judgement in the selection of a model. Here we emphasize

the Tucker-Lewis index (TLI), normed fit index (NFI), comparative fit index (CFI), and

the root mean square error of approximation (RMSEA) index to evaluate goodness of fit,

but also to present the chi-squared test statistic. Although there are no exact standards for

which fit indexes to use or their values, typical guidelines are that the TLI, CFI, and NFI

should be greater than .9 and the RMSEA should be less than .05. The reason for

selecting TLI and RMSEA is because they account for parsimony. Higher Indexes for

TLI and RMSEA would indicate a more parsimonious model as illustrated by the ratio

between the degrees of freedom in the Target model as compared to the Null model. TLI

was used despite it being the accepted index for nested models even though these models

are not nested because of its added information regarding parsimony (Hu & Bentler,

1995).

INSERT TABLE 3 HERE

Model 1 (See Figure 2). The construct of Graphic Display was assessed using the

SEM model. Vet Club items 6,7,8, 9, and Spring Benchmark items 4 and 10 were used

as the measured variables. The fit indexes (NFI, TLI, CFI) were all in acceptable ranges

indicating that data fit the theoretical model (See Table 3).

INSERT FIGURE 2 HERE

Model 2 (See Figure 3). Reading Simple Graphs was assessed using the SEM

model and the measured variables from Vet Club 1 and 2, and Spring Benchmarks 20, 23,

40, and 47. The RMSEA of .077 indicated a moderate fit. Application in research

indicated that values of .05 or less indicate a close fit to the theory in relation to degrees

of freedom. Values bounded by .05 and .08 indicate a reasonable error of approximation

and values greater than this indicate a poor model fit with respect to degrees of freedom

(cf. Browne & Cudeck, 1993). The NFI, TLI, CFI, were .984, .983, and .993,

respectively indicating a reasonable fit for the data as shown in Table 3.

INSERT FIGURE 3 HERE

Model 3 (See Figure 4). The construct of representation of a variety of

relationships was assessed using the SEM model. Similar to Model 2, the RMSEA was a

.078 indicating a moderate fit in terms of degrees of freedom. However, the CFI, TLI,

and NFI all indicated a reasonable fit for the model as shown in Table 3.

INSERT FIGURE 4 HERE

Model 4 (See Figure 5). Presented a case for concern. The number of measured

variables, all single items, provided rather dubious results (See Table 3). One concern

could be the 2 degrees of freedom lends itself to a just identified model. Future attempts

at providing evidence of this theory should include a minimum of four composite

variables.

INSERT FIGURE 5 HERE

Discussion

The development of understanding of specific mathematics topics such as graphic

representation, as implemented by standards-based materials, takes place over a two to

three-year time span. An essential part of developing understanding is to build on

students' prior knowledge. If that construction of knowledge is to be successful, it is essential to know the structure of the knowledge and how it relates to the target learning goal as stated in a mathematics standard or benchmark. The results of this study provide a first prototype for the type of assessment alignment and data analysis that can be used to develop models for students' knowledge construction.

The first step in examining knowledge construction is to propose a "map" of student learning for specific important content areas. The maps from the *Atlas for Science Literacy* (AAAS, 2001) provide a helpful starting point. In testing the fit of such a map with actual student knowledge, the ideal approach would be to construct a test that consists of items aligned with components in the map, rather than selecting items *post hoc* the way it was done in this study. Nevertheless, the results from the factor analysis indicate that we were able to identify items from a variety of assessments that did align with the map and that could be used to characterize student prior knowledge in specific areas of graphic representation. Furthermore, the four components reflected by the modified *Atlas* map accounted for nearly half the variance in student understanding of graphic representation, as measured by the Texas Assessment of Academic Skills (TAAS).

The Structural Equation Modeling (SEM) approach appears to be a promising way to test learning maps for their fit with student knowledge construction and to examine the complexity of this knowledge. Each of the four SEM models provided a strong confirmation that the measures we used did indeed represent students' knowledge of the construct that was identified.

In this study, because of the small n, we were only able to test whether each of the four individual constructs provided a reasonable representation of the map component. We were not able to put the four components together to examine their interrelationships and their connection to the final target learning goal for graphic representation. A much larger sample size, along with combining some of the correlated measures within each factor would be necessary for accomplishing this analysis. Our future research will include expanding the sample so that a full SEM analysis can be conducted in order to examine the complexities and interrelationships of prior knowledge components. We also plan to extend the work to other important mathematical topics, including rational number and algebraic concepts of change and relationship between two variables.

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Table 1. Summary of Regression Analysis for Variables Predicting TAAS Score (n =82)

	Unstandardized Coefficients					
	B	Std. Error	Beta	Rs^2	t	Sig.
Constant	28.099	5.364			5.239	.000
VC 1	-17.335	8.612	-.449	.376	-2.103	.049
VC 2	25.837	7.494	.768	.547	3.448	.001
VC 3	2.834	2.030	.170	.382	1.397	.168
VC 4	2.392	2.603	.126	.097	.919	.362
VC 5	-5.284	2.685	-.272	-.139	-1.968	.054
VC 6	.992	1.370	.129	.263	.724	.472
VC 7	2.553	2.697	.124	.127	.947	.348
VC 8	2.321	3.174	.149	.358	.731	.468
VC 9	-.289	3.326	-.148	.214	-.688	.494
FB 4	-.489	1.781	-.032	.167	-.275	.784
SB 9	3.538	1.643	.230	.301	2.154	.036
SB 10	3.093	1.796	.199	.248	1.722	.091
SB 12	1.168	1.663	.075	.118	.702	.485
SB 20	.193	1.873	.011	-.028	.103	.918
SB 23	1.620	1.674	.104	.162	.967	.338
SB 40	1.492	2.001	.084	.197	.746	.459
SB 47	1.982	1.826	.129	.433	1.085	.283
SB 56	1.222	1.779	.076	.375	.687	.495

Note. R^2 = .465; p = .003

Table 2. Factor Structure Matrix for Variables Predicting TAAS Score (n =82)

	Component				
Variable	I	II	III	IV	h²
VETCLUB1	.007	.870	.189	-.224	.862
VETCLUB2	.045	.875	.152	-.164	.828
SB20	-.018	.557	-.009	-.127	.060
SB23	-.067	-.655	.139	-.035	.084
SB40	.201	.375	-.038	.014	-.012
SB47	.405	.526	.146	.174	.223
VETCLUB3	.035	.294	.660	.343	.075
VETCLUB4	.153	.125	.756	.046	.171
SB56	.363	.035	.412	.157	.324
VETCLUB6	.795	-.054	.146	-.208	-.087
VETCLUB7	.520	.158	.002	.280	-.070
VETCLUB8	.912	.163	.170	.09	-.158
VETCLUB9	.910	.121	.170	-.065	.042
SB4	.412	.313	-.402	.092	-.038
SB10	.538	-.138	-.045	-.184	-.039
VETCLUB5	.248	.088	-.062	.801	.056
SB9	-.015	.148	-.038	-.330	-.162
SB12	-.009	.037	.019	-.391	-.030
Trace	.712	1.597	.003	-.186	2.126
% of Variance	18.1	11.8	9.8	8.3	48

Note. Principal Component Analysis with a Promax with Kaiser Normalization. Coefficients greater than |.33| are underlined.

Table 3   Fit Indexes for SEM Models

Model	χ^2	df	p	CFI	TLI	NFI	RMSEA
Model 1	26.726	9	.002	.978	.948	.967	.016
Model 2	16.452	9	.058	.993	.983	.984	.077
Model 3	4.646	2	.098	.985	.954	.974	.078
Model 4	.183	2	.912	1.00	1.20	.999	.005

Note. $n = 140$
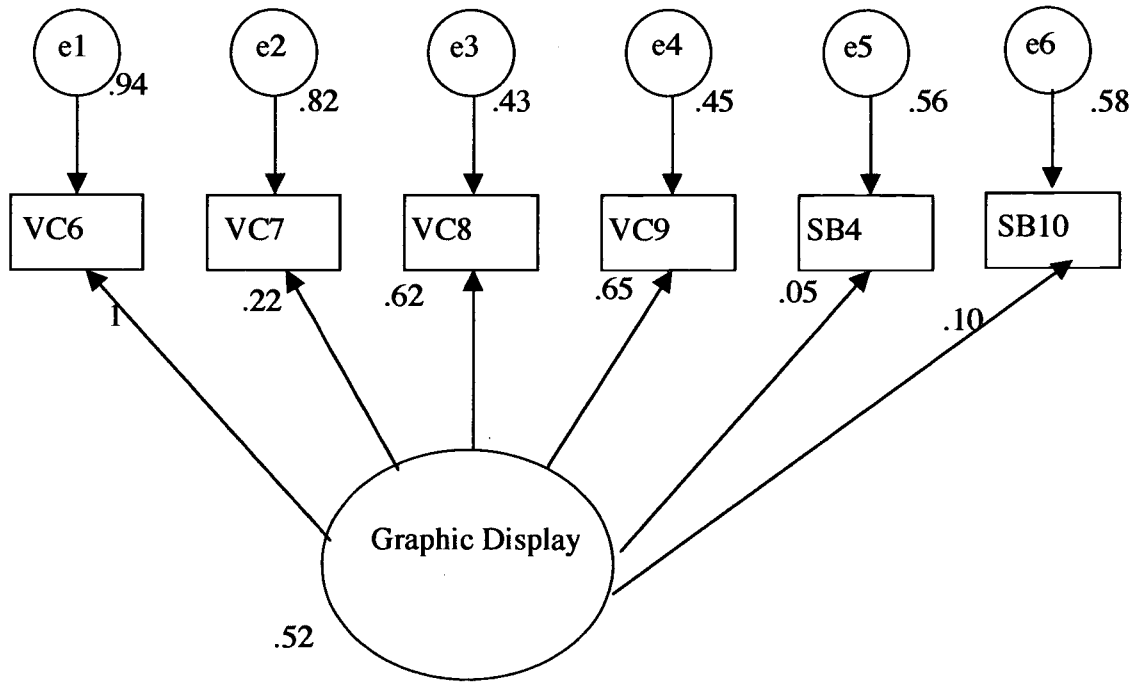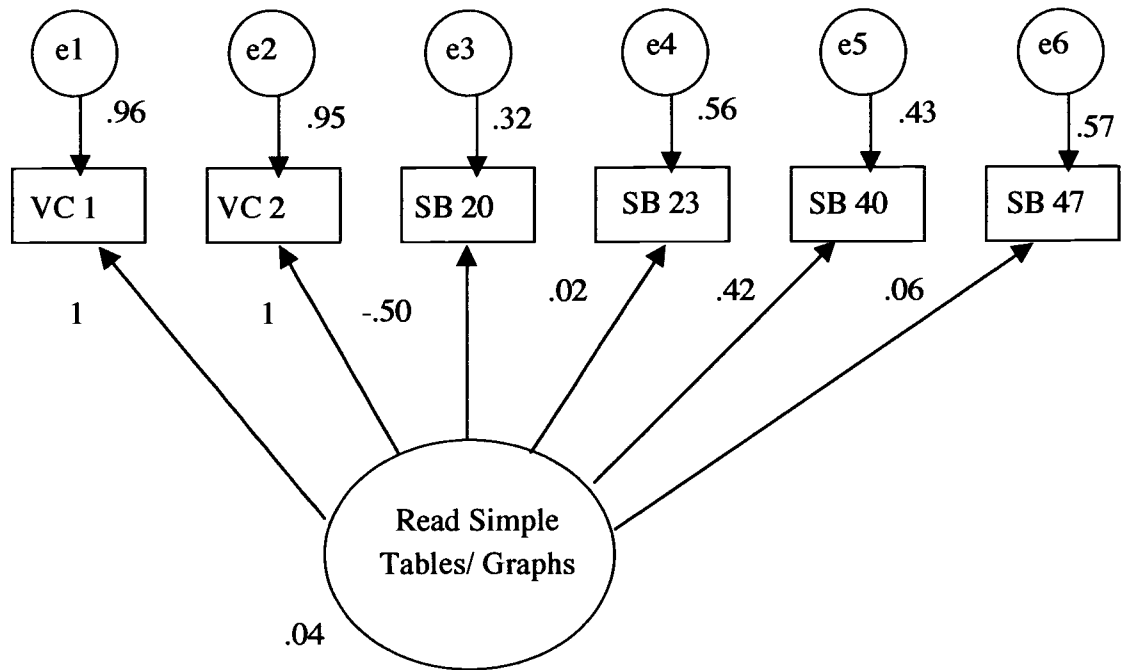
Figure 2. SEM Model #1, Graphic Display.

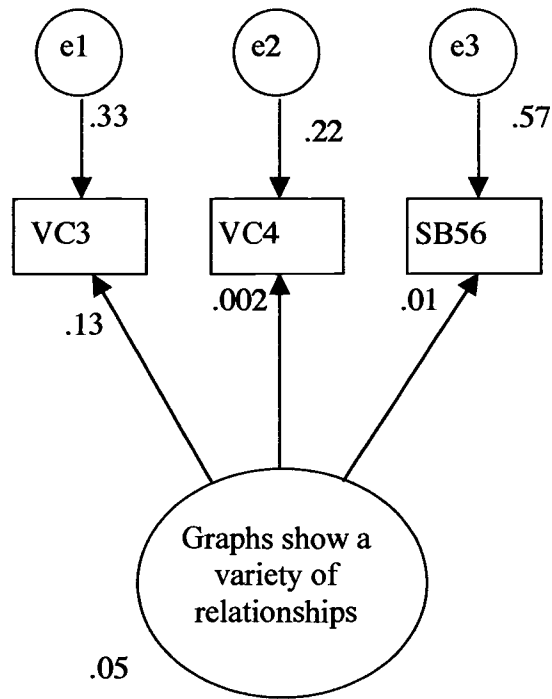Figure 3. SEM Model #2, Read Simple Tables/Graphs.

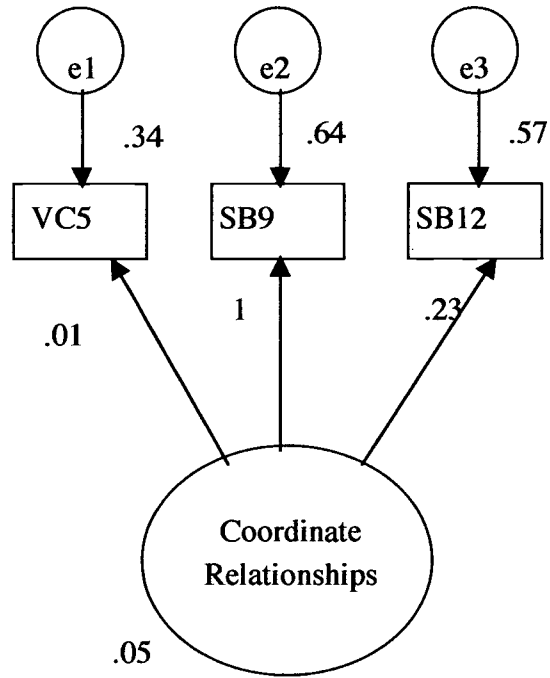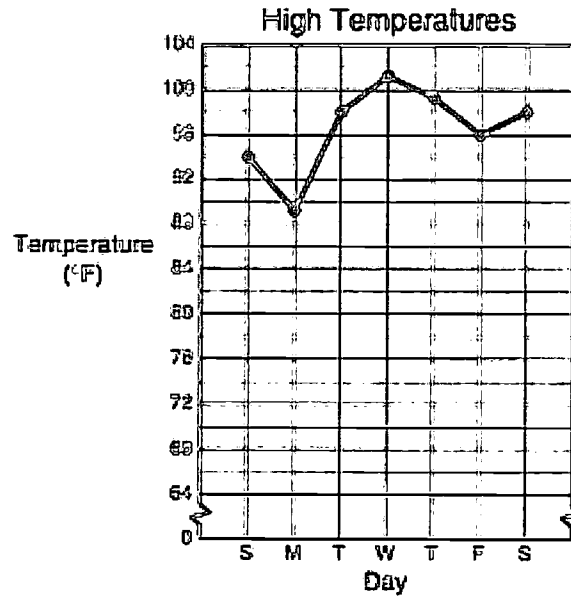Figure 4. SEM Model #3, Graphs Show a Variety of Relationships.

Figure 5. SEM Model #4, Coordinate Relationships.

Appendix A
Spring Benchmark#1 (Factor I)

10 The graph shows the daily high temperatures in degrees Fahrenheit for 1 week in August.

High Temperatures

Which table shows the same information as the graph?

High Temperatures

F

Day	S	M	T	W	T	F	S
Temperature (°F)	95	89	99	102	98	94	97

High Temperatures

G

Day	S	M	T	W	T	F	S
Temperature (°F)	94	89	98	101	99	96	98

High Temperatures

H

Day	S	M	T	W	T	F	S
Temperature (°F)	93	89	97	102	97	95	97

High Temperatures

J

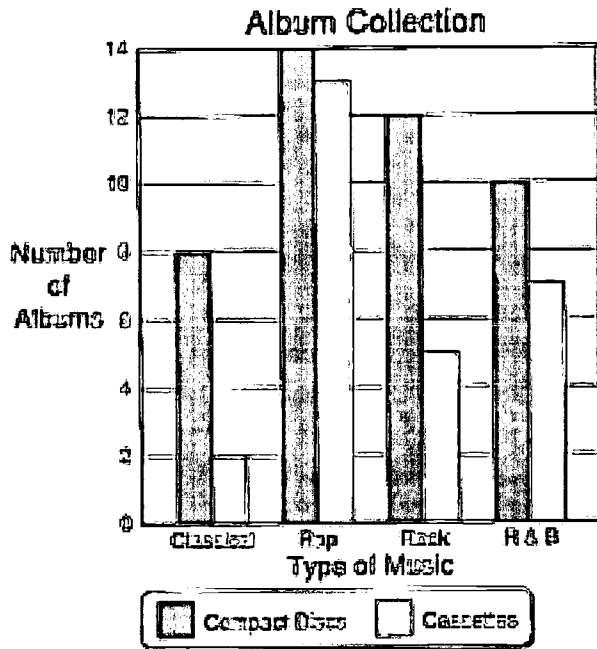| Day	S	M	T	W	T	F	S
Temperature (°F)	94	89	93	100	98	96	97

Appendix B
Spring Benchmark Item #23 (Factor II)

23 Angelo has a collection of music albums. The
    graph describes Angelo's album collection by
    style of music. It also shows how many
    albums are on compact disc and how many
    are on cassette.

Album Collection

How many compact discs does Angelo own
in all?

A 71

B 61

C 34

D 27

E Not Here

Appendix C
Spring Benchmark #56 (Factor III)

58 The chart shows 3 brands of candles and how
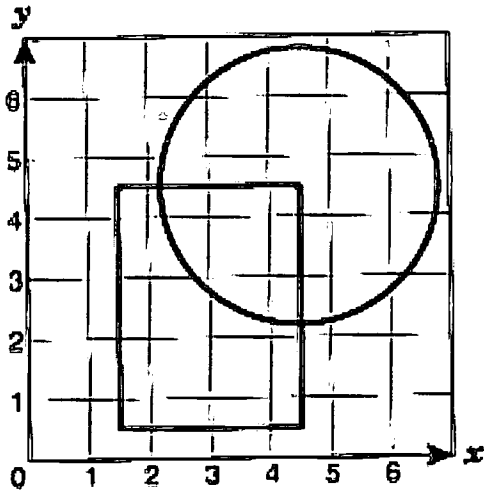   many millimeters each candle burns per hour.

Candle Burn Rate

Brand of Candle	Length Burned per Hour
A	35 mm
B	49 mm
C	36 mm

According to this information, how many
millimeters would Brand C burn in 4 hours?

F   9 mm

G   108 mm

H   124 mm

J   144 mm

K   Not Here

31

Appendix D
Spring Benchmark # 9 (Factor IV)

9   A circle and a rectangle are shown on the grid.

Which coordinate pair is located inside the
circle and outside the rectangle?

A   (2, 5)

B   (3, 4)

C   (4, 4)

D   (5, 3)

Appendix E
Vet Club Tasks* and Sample Student Response

1. Your job is to prepare a graph to go with Jenny's article. Organize the information from her notes into a graph that will show how many of her friends have no pets, one pet, two pets, and so on.

VC 1. Students are able to draw a graph

VC 2. Students are able to draw graphs that contain an x and y axis

VC 3. Students are able to label each axis appropriately

VC 4. Students list "number of people" as a category

VC 5. Students are able to correctly plot values on a graph

2. What number should Jenny put in the blank? _____

VC 6. Students are able to interpret information on the graph and apply the information to a real-world situation

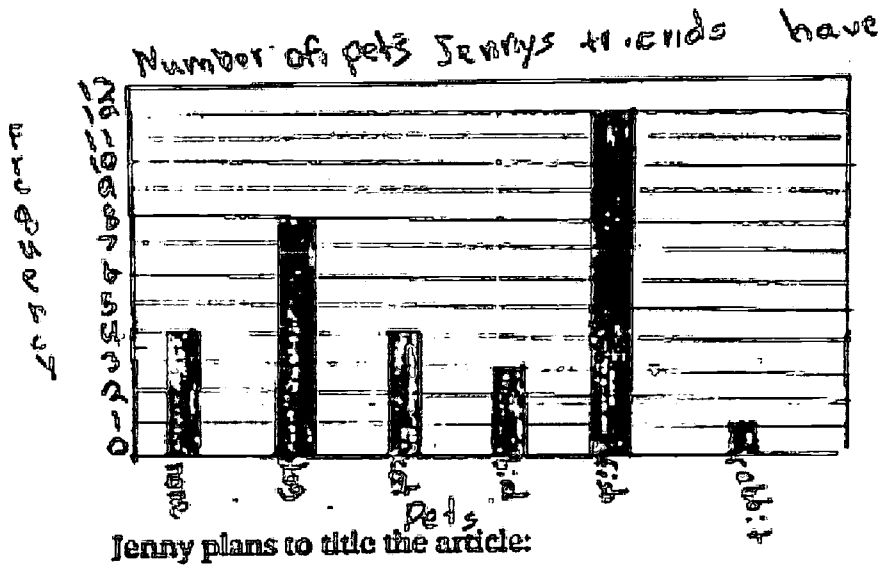3. Explain why the number you chose is the best number to complete the headline.

VC 7. Students are able to explain how the data is connected to the graph

VC 8. Students are able to connect their explanation to measures of central tendency

VC 9. Students are able to refer to and specifically use a measure of central tendency

*Each task is scored 0 or 1.

Appendix F Sample Student Response

Number of pets Jennys friends have

Jenny plans to title the article:

Typical Future Veterinarians Club
Member Has _7_ House Pets

2. What number should Jenny put in the blank? __2__

3. Explain why the number you chose is the best number to
   complete the headline. I chose 2, because it
   is the average number of pets

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